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SWITCHING INVARIANT COMMON MINIMAL DOMINATING SYMMETRIC n-SIGRAPHS

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Abstract

In this paper, we define the common minimal dominating symmetric n-sigraph of a given symmetric n-sigraph and offer a structural characterization of common minimal dominating symmetric n-sigraphs. In the sequel, we also obtained switching equivalence characterizations $\overline{S_n} \sim CMD(S_n)$ and $CMD(S_n) \sim N(S_n)$ where S_n, $CMD(S_n)$ and $N(S_n)$ are complementary symmetric n-sigraph, common minimal dominating symmetric n-sigraph and neighborhood symmetric n-sigraph of a symmetric n-sigraph S_n respectively.

1. Introduction

Unless mentioned or defined otherwise, for all terminology and notion in graph theory the reader is refer to [1]. We consider only finite, simple graphs free from self-loops. Let $n \geq 1$ be an integer. An *n-tuple* $(a_1, a_2, ..., a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, ..., a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of

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all symmetric n-tuples. Note that H_n is a group under coordinate wise multiplication, and the order of H_n is 2^m, where $m = \lceil \frac{n}{2} \rceil$.

A *symmetric n-sigraph (symmetric n-marked graph)* is an ordered pair $S_n = (G, \sigma)$ $(S_n = (G, \mu))$, where $G = (V, E)$ is a graph called the *underlying graph* of S_n and $\sigma : E \to H_n$ $(\mu : V \to H_n)$ is a function.

In this paper by an *n-tuple/n-sigraph/n-marked graph* we always mean a symmetric n-tuple/symmetric n-sigraph/symmetric n-marked graph.

An n-tuple $(a_1, a_2, ..., a_n)$ is the *identity n-tuple*, if $a_k = +$, for $1 \leq k \leq n$, otherwise it is a *non-identity n-tuple*. In an n-sigraph $S_n = (G, \sigma)$ an edge labelled with the identity n-tuple is called an *identity edge*, otherwise it is a *non-identity edge*.

Further, in an n-sigraph $S_n = (G, \sigma)$, for any $A \subseteq E(G)$ the *n-tuple* $\sigma(A)$ is the product of the n-tuples on the edges of A.

In [10], the authors defined two notions of balance in n-sigraph $S_n = (G, \sigma)$ as follows (See also R. Rangarajan and P.S.K.Reddy [6]:

Definition : Let $S_n = (G, \sigma)$ be an n-sigraph. Then,

(i) S_n is *identity balanced* (or *i-balanced*), if product of n-tuples on each cycle of S_n is the identity n-tuple, and

(ii) S_n is *balanced*, if every cycle in S_n contains an even number of non-identity edges.

Note: An i-balanced n-sigraph need not be balanced and conversely.

The following characterization of i-balanced n-sigraphs is obtained in [10].

Theorem 1.1 : ***(E. Sampathkumar et al. [10])*** : An n-sigraph $S_n = (G, \sigma)$ is i-balanced if, and only if, it is possible to assign n-tuples to its vertices such that the n-tuple of each edge uv is equal to the product of the n-tuples of u and v.

In [10], the authors also have defined switching and cycle isomorphism of an n-sigraph $S_n = (G, \sigma)$ as follows: (See also [4], [7-9], [12-17], [19-23]).

Let $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$, be two n-sigraphs. Then S_n and S'_n are said to be *isomorphic*, if there exists an isomorphism $\phi : G \to G'$ such that if uv is an edge in S_n with label $(a_1, a_2, ..., a_n)$ then $\phi(u)\phi(v)$ is an edge in S'_n with label $(a_1, a_2, ..., a_n)$.

Given an n-marking μ of an n-sigraph $S_n = (G, \sigma)$, *switching* S_n with respect to μ is the operation of changing the n-tuple of every edge uv of S_n by $\mu(u)\sigma(uv)\mu(v)$. The n-

sigraph obtained in this way is denoted by $\mathcal{S}_\mu(S_n)$ and is called the *μ-switched n-sigraph* or just *switched n-sigraph*.

Further, an n-sigraph S_n *switches* to n-sigraph S'_n (or that they are *switching equivalent* to each other), written as $S_n \sim S'_n$, whenever there exists an n-marking of S_n such that $\mathcal{S}_\mu(S_n) \cong S'_n$.

Two n-sigraphs $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$ are said to be *cycle isomorphic*, if there exists an isomorphism $\phi : G \to G'$ such that the n-tuple $\sigma(C)$ of every cycle C in S_n equals to the n-tuple $\sigma(\phi(C))$ in S'_n.

We make use of the following known result (see [10]).

Theorem 1.2 ***(E. Sampathkumar et al. [10])*** : Given a graph G, any two n-sigraphs with G as underlying graph are switching equivalent if, and only if, they are cycle isomorphic.

Let $S_n = (G, \sigma)$ be an n-sigraph. Consider the n-marking μ on vertices of S defined as follows: each vertex $v \in V$, $\mu(v)$ is the product of the n-tuples on the edges incident at v. *Complement* of S is an n-sigraph $\overline{S_n} = (\overline{G}, \sigma')$, where for any edge $e = uv \in \overline{G}$, $\sigma'(uv) = \mu(u)\mu(v)$. Clearly, $\overline{S_n}$ as defined here is an i-balanced n-sigraph due to Theorem 1.1.

2. Common Minimal Dominating n-Sigraph of an n-Sigraph

Let $G = (V, E)$ be a graph. A set $D \subseteq V$ is a dominating set of G, if every vertex in $V - D$ is adjacent to some vertex in D. A dominating set D of G is minimal, if for any vertex $v \in D$, $D - \{v\}$ is not a dominating set of G (See, [5]).

Kulli and Janakiram [2] introduced a new class of intersection graphs in the field of domination theory. The common minimal dominating graph $CMD(G)$ of a graph G is the graph having same vertex set as G with two vertices adjacent in $CMD(G)$ if, and only if, there exists a minimal dominating set in G containing them.

In this paper, we introduce a natural extension of the notion of common minimal dominating graph to the realm of n-sigraphs since this appears to have interesting connections with complementary n-sigraph and neighborhood n-sigraph.

The *common minimal dominating n-sigraph* $CMD(S_n)$ of an n-sigraph $S_n = (G, \sigma)$ is an n-sigraph whose underlying graph is $CMD(G)$ and the n-tuple of any edge uv in $CMD(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n. Further, an n-sigraph

$S_n = (G, \sigma)$ is called common minimal dominating n-sigraph, if $S_n \cong CMD(S'_n)$ for some n-sigraph S'_n. In this paper we will give a structural characterization of n-sigraphs which are common minimal dominating n-sigraphs.

The following result indicates the limitations of the notion $CMD(S_n)$ introduced above, since the entire class of i-unbalanced n-sigraphs is forbidden to be common minimal dominating n-sigraphs.

Theorem 2.1 : For any n-sigraph $S_n = (G, \sigma)$, its common minimal dominating n-sigraph $CMD(S_n)$ is i-balanced.

Proof : Since the n-tuple of any edge uv in $CMD(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n, by Theorem 1.1, $CMD(S_n)$ is i-balanced. □

For any positive integer k, the k^{th} iterated common minimal dominating n-sigraph $CMD(S_n)$ of S_n is defined as follows:

$$(CMD)^0(S_n) = S_n, \ (CMD)^k(S_n) = CMD((CMD)^{k-1}(S_n)).$$

Corollary 2.2 : For any n-sigraph $S_n = (G, \sigma)$ and any positive integer k, $(CMD)^k(S_n)$ is i-balanced.

The following result characterize n-sigraphs which are common minimal dominating n-sigraphs.

Theorem 2.3 : An n-sigraph $S_n = (G, \sigma)$ is a common minimal dominating n-sigraph if, and only if, S_n is i-balanced n-sigraph and its underlying graph G is a common minimal dominating graph.

Proof : Suppose that S_n is i-balanced and G is a $CMD(G)$. Then there exists a graph H such that $CMD(H) \cong G$. Since S_n is i-balanced, by Theorem 1.1, there exists an n-marking μ of G such that each edge uv in S_n satisfies $\sigma(uv) = \mu(u)\mu(v)$. Now consider the n-sigraph $S'_n = (H, \sigma')$, where for any edge e in H, $\sigma'(e)$ is the n-marking of the corresponding vertex in G. Then clearly, $CMD(S'_n) \cong S_n$. Hence S_n is a common minimal dominating n-sigraph.

Conversely, suppose that $S_n = (G, \sigma)$ is a common minimal dominating n-sigraph. Then there exists an n-sigraph $S'_n = (H, \sigma')$ such that $CMD(S'_n) \cong S_n$. Hence G is the $CMD(G)$ of H and by Theorem 2.1, S_n is i-balanced. □

In [2], the authors characterized graphs for which $CMD(G) \cong \overline{G}$.

Theorem 2.4 ***(Kulli and Janakiram [1])*** : For any graph $G = (V, E)$, $CMD(G) \cong \overline{G}$ if, and only if, every minimal dominating set of G is independent.

We now characterize n-sigraphs whose common minimal dominating n-sigraphs and complementary n-sigraphs are switching equivalent.

Theorem 2.5: For any n-sigraph $S_n = (G, \sigma)$, $\overline{S_n} \sim CMD(S_n)$ if, and only if, every minimal dominating set of G is independent.

Proof : Suppose $\overline{S_n} \sim CMD(S_n)$. This implies, $\overline{S_n} \cong CMD(S_n)$ and hence by Theorem 2.4, every minimal dominating set of G is independent.

Conversely, suppose that every minimal dominating set of G is independent. Then $\overline{S_n} \cong CMD(S_n)$ by Proposition 2.4. Now, if S_n is an n-sigraph with every minimal dominating set of underlying graph G is independent, by the definition of complementary n-sigraph and Theorem 2.1, $\overline{S_n}$ and $CMD(S_n)$ are i-balanced and hence, the result follows from Theorem 1.2. □

In [18], the authors introduced neighborhood n-sigraph of an n-sigraph as follows:

The *neighborhood n-sigraph* $N(S_n)$ of an n-sigraph $S_n = (G, \sigma)$ is an n-sigraph whose underlying graph is $N(G)$ and the n-tuple of any edge uv in $N(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n. Further, an n-sigraph $S_n = (G, \sigma)$ is called neighborhood n-sigraph, if $S_n \cong N(S'_n)$ for some n-sigraph S'_n. The following result restricts the class of neighborhood graphs.

Theorem 2.6 ***(Rangarajan et al. [8])*** : For any n-sigraph $S_n = (G, \sigma)$, its neighborhood n-sigraph $N(S_n)$ is i-balanced.

We now characterize n-sigraphs whose common minimal dominating n-sigraphs and neighborhood n-sigraphs are switching equivalent. In case of graphs the following result is due to Kulli and Janakiram [3].

Theorem 2.7 ***(Kulli and Janakiram [3])*** : If G is a $(p-2)$-regular graph with $p \geq 6$, then $CMD(G) \cong N(G)$.

Theorem 2.8 : For any n-sigraph $S_n = (G, \sigma)$, $CMD(S_n) \sim N(S_n)$ if, and only if, G is a $(p-2)$-regular graph with $p \geq 6$.

Proof : Suppose $CMD(S_n) \sim N(S_n)$. This implies, $CMD(G) \cong N(G)$ and hence by Theorem 2.7, we see that the graph G must be $(p-2)$-regular graph with $p \geq 6$.

Conversely, suppose that G is $(p-2)$-regular graph with $p \geq 6$. Then $CMD(G) \cong N(G)$ by Proposition 2.7. Now, if S_n is an n-sigraph with underlying graph as $(p-2)$-regular graph with $p \geq 6$, by Theorems 2.1 and 2.6, $CMD(S_n)$ and $N(S_n)$ are i-balanced and hence, the result follows from Theorem 1.2. □

Theorem 2.9: For any two n-sigraphs S_n and S'_n with the same underlying graph, their common minimal dominating n-sigraphs are switching equivalent.

Proof : Suppose $Sn = (G, \sigma)$ and $S'_n = (G', \sigma')$ be two n-sigraphs with $G \cong G'$. By Theorem 2.1, $CMD(S_n)$ and $CMD(S'_n)$ are i-balanced and hence, the result follows from Theorem 1.2. □

3. Complementation

In this section, we investigate the notion of complementation of a graph whose edges have signs (a *sigraph*) in the more general context of graphs with multiple signs on their edges. We look at two kinds of complementation: complementing some or all of the signs, and reversing the order of the signs on each edge.

For any $m \in H_n$, the *m-complement* of $a = (a_1, a_2, ..., a_n)$ is: $a^m = am$. For any $M \subseteq H_n$, and $m \in H_n$, the *m-complement* of M is $M^m = \{a^m : a \in M\}$.

For any $m \in H_n$, the *m-complement* of an n-sigraph $S_n = (G, \sigma)$, written (S_n^m), is the same graph but with each edge label $a = (a_1, a_2, ..., a_n)$ replaced by a^m.

For an n-sigraph $S_n = (G, \sigma)$, the $CMD(S_n)$ is i-balanced. We now examine, the condition under which m-complement of $CMD(S_n)$ is i-balanced, where for any $m \in H_n$. For an n-sigraph $S_n = (G, \sigma)$, the $CMD(S_n)$ is i-balanced. We now examine, the conditions under which m-complement of $CMD(S_n)$ is i-balanced, where for any $m \in H_n$.

Theorem 3.1 : Let $S_n = (G, \sigma)$ be an n-sigraph. Then, for any $m \in H_n$, if $CMD(G)$ is bipartite then $(CMD(S_n))^m$ is i-balanced.

Proof : Since, by Theorem 2.1, $CMD(S_n)$ is i-balanced, for each k, $1 \leq k \leq n$, the number of n-tuples on any cycle C in $CMD(S_n)$ whose k^{th} co-ordinate are $-$ is even. Also, since $CMD(G)$ is bipartite, all cycles have even length; thus, for each k, $1 \leq k \leq n$, the number of n-tuples on any cycle C in $CMD(S_n)$ whose k^{th} co-ordinate are $+$ is also even. This implies that the same thing is true in any m-complement, where for any $m, \in H_n$. Hence $(CMD(S_n))^t$ is i-balanced. □

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